

## Game Theory

*Strategic thinking is the art of outdoing an adversary, knowing that the adversary is trying to do the same to you.*

Dixit and Nalebuff

*Game theory* is used to explain how two or more players make decisions or choose actions when their actions (or strategies) affect each participant. Each player determines his or her best response to the possible actions of every other player. According to game theory, a player's choice of strategy depends on the strategy the player thinks other players will choose. In some cases, these strategies reinforce each other, but in other cases, they do not. When the chosen strategies reinforce each other, the game achieves what is called a *Nash Equilibrium*. The Nash Equilibrium is named after John F. Nash, Jr., who was co-winner of the 1994 Nobel Prize in Economics for his work in this area and the subject of the 2001 movie *A Beautiful Mind*.

Game theory provides insights into how business and government decisions are made and has numerous real-world applications. For example, game theory has helped economists analyze antitrust policy, tariff wars and auctioning behavior. This lesson is an introduction to the basic elements of game theory. As you do the math, think about the implications of the results.

### Part A

#### The Basic Elements of Game Theory

The three basic elements of a game are

- (A) the players,
- (B) the strategies available to each player,
- (C) the payoffs each player receives.

These three elements are summarized in a table called a *payoff matrix*. A payoff matrix describes the payoffs to each player for combinations of given strategies. Here is an example of a payoff matrix:

		Coke	
		Advertise	Don't Advertise
Pepsi	Advertise	80, 80	120, 45
	Don't Advertise	45, 120	100, 100

The first number in each square refers to the payoff for the row (horizontal) player, here Pepsi. The second number in each square refers to the payoff for the column (vertical) player, here Coke. The numbers represent the profit for Pepsi and Coke.

In this game:

- (A) The players are Pepsi and Coke.
- (B) The strategies available to each player:
  - Pepsi, as the row player, can choose either Advertise or Don't Advertise.
  - Coke, as the column player, can choose either Advertise or Don't Advertise.

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(C) The payoffs each player receives:

- If Pepsi chooses Advertise and Coke chooses Advertise, Pepsi earns 80 and Coke earns 80.
- If Pepsi chooses Advertise and Coke chooses Don't Advertise, Pepsi earns 120 and Coke earns 45.
- If Pepsi chooses Don't Advertise and Coke chooses Advertise, Pepsi earns 45 and Coke earns 120.
- If Pepsi chooses Don't Advertise and Coke chooses Don't Advertise, Pepsi earns 100 and Coke earns 100.

Each player gains a lot from advertising when the other player does not advertise because the advertiser gains a larger share of the market. If both advertise, the gain is less than if both don't advertise because advertising costs money.

In some games, one player will have a *dominant strategy*. A dominant strategy is the best strategy for one player regardless of the strategy the other player follows. In the game with Pepsi and Coke, the dominant strategy for Pepsi is to choose Advertise. This is because Pepsi earns more regardless of which strategy Coke chooses.

- If Coke chooses Advertise, Pepsi earns 80 choosing Advertise, which is greater than earning 45 from choosing Don't Advertise.
- If Coke chooses Don't Advertise, Pepsi earns 120 choosing Advertise, which is greater than earning 100 from choosing Don't Advertise.

The dominant strategy for Coke is to choose Advertise. This is because Coke earns more regardless of what Pepsi chooses.

- If Pepsi chooses Advertise, Coke earns 80 choosing Advertise, which is greater than earning 45 from choosing Don't Advertise.
- If Pepsi chooses Don't Advertise, Coke earns 120 choosing Advertise, which is greater than earning 100 from choosing Don't Advertise.

A *dominated strategy* yields a lower payoff than at least one other strategy. In this game, the dominated strategy for Pepsi is Don't Advertise; it is dominated by Advertise. Regardless of the strategy selected by Coke, Pepsi gains more by choosing Advertise. If Pepsi chooses Don't Advertise, the payoff is 45, while a strategy of Advertise has a payoff of 80. Since 45 is less than 80, the dominated strategy is Don't Advertise.

The dominated strategy for Coke is Don't Advertise; it is dominated by Advertise. If Coke chooses Don't Advertise, Coke receives 45 if Pepsi chooses Advertise and 100 if Pepsi chooses Don't Advertise. Since 45 is less than 100, the dominated strategy for Coke is Don't Advertise.

A Nash Equilibrium is a combination of strategies for each player, such that each chooses his or her best response to the other's strategy choice. In this game, the Nash Equilibrium is Pepsi choosing Advertise and Coke choosing Advertise. Although in this example both Coke and Pepsi select the same strategy, in a Nash Equilibrium the players do not have to select the same strategy.

If Coke chooses Advertise, Pepsi is better off choosing Advertise (80 compared with 45). If Coke chooses Don't Advertise, Pepsi is better off choosing Advertise (120 compared with 100). So whatever

strategy Coke chooses, Pepsi is always better off choosing Advertise. Note that Pepsi has no incentive to choose any strategy other than Advertise.

Likewise if Pepsi chooses Advertise, then Coke is always better off choosing Advertise (80 compared with 45). If Pepsi chooses Don't Advertise, Coke is always better off choosing Advertise (120 compared with 100). So whatever strategy Pepsi chooses, Coke is always better off choosing Advertise. Note that Coke has no incentive to choose any strategy other than Advertise.

A Nash Equilibrium is similar to a market equilibrium in that there is no incentive for producers and consumers to change from the equilibrium price. Thus a Nash Equilibrium is an "enforceable" equilibrium because the firms do not have an incentive to cheat as they might in a cartel.

Other economic examples of game-theory applications are decisions by firms about what price to charge, whether to enter a market, where to locate and what kind of product or quality level to produce; decisions by a central bank on monetary policy actions and decisions by a nation on the optimal tariff policy.

**Part B**

**The Prisoner’s Dilemma Game**

One classic type of game is the *prisoner’s dilemma game*. Prisoner’s dilemma games are games in which each player has a dominant strategy; and when both players play the dominant strategy, the payoffs are smaller than if each player played the dominated strategy. The dilemma is how to avoid this bad outcome.

The basics of the prisoner’s dilemma game are as follows: Two prisoners have the option to confess or not confess to a crime they committed. The prosecutor has only enough information to convict both criminals of a minor offense and is, therefore, relying on a confession. The minor offense carries one year in jail. The prisoners are questioned in different cells, without the ability to communicate. They are told that if one prisoner confesses while the other remains silent, the prisoner confessing will go free and the prisoner remaining silent will serve 20 years in jail. If both prisoners confess, both prisoners will serve three years in jail.

If a player goes free, the payoff is 0. If a player serves one year in jail, the payoff is  $-1$ . If a player spends 20 years in jail, the payoff is  $-20$ . Use these numbers in your payoff matrix. Note that the negative numbers come from losing years of freedom.

1. Determine the three basic elements of the game.
  - (A) The players:
  - (B) The strategies for each player:
  - (C) The payoffs for each player:
  
2. Create a payoff matrix for the prisoner’s dilemma game.


3. Identify any dominant strategies.
  
4. Identify any dominated strategies.
  
5. Find the Nash Equilibrium.

**Part C**

**Variation of the Prisoner’s Dilemma Game**

You are in a class with one other student. It is the end of the semester, and final exams are in a week. Your teacher has said the final exam will be graded so that anyone who scores the class average on the final exam will receive a “B” in the class. Anyone who scores above the average will receive an “A” in the class, and anyone who scores below the average will fail the class. You would certainly score higher on the exam than the other student. You and the other student have made an agreement not to take the final exam so that the class average is zero and you both receive “B” grades.

- 6. Determine the three basic elements of the game.
  - (A) The players:
  - (B) The strategies for each player:
  - (C) The payoffs for each player:

- 7. Create a payoff matrix for this game.


- 8. What is your dominant strategy? (Underline the correct answer)  
*Take the Exam or Not Take the Exam*
- 9. Using a four-point scale (A = 4, B = 3, C = 2 and D = 1), which choice results in the highest class GPA?

If you finished Parts B and C correctly, you will realize that when each player chooses his or her dominant strategy, the result is unattractive to the group.

The key to avoiding the prisoner’s dilemma outcome of lower payoffs for both players is to find a way for players to credibly commit to playing a dominated strategy. Merely having both prisoners agree to Not Confess or both students to Not Take the Exam will not work. This results because it is always optimal for Prisoner 1 (or Prisoner 2) to still play the Confess strategy, and it is always optimal

